A.J.O. Cruickshank University of Dundee Dundee, Scotland A.F. Anderson International Research and Development Company Ltd Newcastle-upon-Tyne, England R.W. Menzies University of Manitoba Winnipeg, Canada

# ABSTRACT

The limits of stability in reluctance machines are known to be narrow, especially in machines of improved design with high reactance ratio. These stability limits can usefully be investigated with the aid of computed and measured freely accelerating torque speed curves. The machine equations are developed using a frame of reference fixed with respect to the rotor. Machine performance is then simulated on an analogue computer. A large number of freely accelerating torque speed curves are then generated for different values of machine parameters. The model is designed to cope simply with different motors and is based on a per unit system. All parameters can be simply varied, so that their effect on performance can be demonstrated. The torque speed curves that result can be said to be characteristic of a machine with given parameters and contain information about starting torque, transient torque dips, synchronisation ability and stability. Since changes in parameters cause marked changes in shape of the freely accelerating torque speed curve, it is possible, by matching the results obtained from a practical test with those from a computer study, to determine where the total performance of the machine lies with respect to the optimum and to decide what changes are necessary to improve design.

### INTRODUCTION

Improved reluctance motors have been available for some time with enhanced synchronous performance, which has been obtained by careful attention to the rotor magnetic circuit and so increasing the ratio of direct to quadrature axis reactance.

Increasing the ratio of the reactance tends to lower the margin of stability and, in order to achieve satisfactory overall design, equal care must be given to the machine electrical circuits, in particular the direct and quadrature axis rotor resistances and the stator resistance.

The dependence of stability on the electrical constants of the machine can be demonstrated on a normally stable machine by increasing the stator resistance to about 2 to 3 times its normal value. The machine will start to exhibit large variations in load angle about synchronous speed. The complete torque speed limit cycle may take place typically at frequencies as high as 17 cycles per second and the magnitude of the speed excursions may be as large as 20% of the synchronous speed. However in a machine with high reactance ratio such instabilities are likely to occur even with normal stator resistance unless the rotor circuits are very carefully designed. Hence it is necessary not only to determine the manner in which certain parameters influence stability, but also to incorporate the knowledge acquired into the design process.

A number of writers have concerned themselves with aspects of reluctance motor stability<sup>1-6</sup>. First, Lipo and Krause<sup>1</sup> derived incremental equations to which the Nyquist criterion was applied in order to determine stability limits. In order to predict the magnitudes of torque speed variations they simulated the dynamic performance of the machine and used the results to aid in the interpretation of the small displacement theory.

Their chief attention was directed to load torque variations and the resultant variation in developed torque and load angle with time. Lawrenson and Bowes<sup>3</sup> have extended Neimark's D-partition method to investigate stability boundaries. The influence of several parameters can be simultaneously displayed if proper choice is made of the abscisse and ordinate, and in doing this the above authors have presented a comprehensive study of stability. Lawrenson, Mathur and Stephenson<sup>5</sup> have also investigated transient performance by simulation on an analogue computer.

The purpose of this paper is to show how freely accelerating torque speed curves can be used to determine the general performance of machines and assist in their design. A particular form of torque curve can be said to be characteristic of a machine with given parameters - a fingerprint of that machine in effect - contained within which is information about starting torque, transient torque dips, synchronising ability, and stability. Since changes in the stator resistance, reactance ratio, rotor direct and quadrature axis resistances all produce marked effects on the shape of the freely accelerating torque speed curve it is possible, by matching the results obtained from the machine under investigation to those obtained from a computer study, to determine where the total performance of the machine lies in relation to the optimum and to decide what changes are necessary in the design.

The method described in this paper is (1) to simulate the machine equations (2) to correlate the simulated with the practical results (3) to draw general conclusions so that the results can be used by any designer of reluctance motors to improve his designs or by a user to estimate where his problems lie.

# REPRESENTATION OF THE MACHINE EQUATIONS BY MEANS OF AN ANALOGUE COMPUTER

In order to render the study as widely applicable as possible equations and results are displayed in per unit form and the base quantities adopted are those of a comparative induction motor of the same active length and frame size as the synchronous reluctance motor. Appendix II tabulates the base quantities which relate to a 4-pole, 50-Hz, 400-V, 3-hp machine with D 100L metric frame size. Stator bore is 4 in (10.2 cm), length 3.25 in (8.25 cm) and outer core diameter 6.5in (16.5 cm). Base speed is synchronous speed in this case 1500 r/min.

The circuit quantities of immediate interest, namely stator resistance  $r_a$ , d-axis resistance  $r_{d2}$ , and q-axis resistance  $r_{q2}$ , are conveniently expressed as  $Sr_1$ ,  $Dr_2$  and  $Qr_2$  where  $r_1$  and  $r_2$  are the stator and referred rotor resistances for the comparative induction motor. By this means the results are related to parameters which may be applied to any design and can be simply related to induction motor results. Parameters expressed in this form can be controlled on an analogue computer by variation of three potentiometers. For the purpose of discussion, two values of reactance ratio are considered corresponding to saturated values of 3.7 and 5.2, which relate to normal voltage for two practical machines which have been built and tested. For each reactance ratio resistances  $r_a$ ,  $r_{d2}$  and  $r_{o2}$ 

Paper T72 049-0, recommended and approved by the Rotating Machinery Committee of the IEEE Power Engineering Society for presentation at the IEEE Winter Meeting, New York, N.Y., January 30-February 4, 1972. Manuscript submitted September 9, 1971; made available for printing November 11, 1971. can be varied over a range of 100:1. The coupled load inertia is either zero or six times rotor inertia and the load torque is set either to zero or to a value corresponding to 3 hp at 1500 r/min. Shaft stiffness is kept constant at the correct value for mild steel.

# Axis Representation and Physical Equations

Figure 1 represents the model for analysis of a reluctance motor. The normal axis transformation is applied to the stator in order to give d and q axis coils fixed to the rotor axes for all instantaneous speeds including zero.



Fig. 1 Moving axes model of reluctance motor

The positive direction of measurement of angle, torque and speed is counter clockwise. The quadrature axis of the rotor is  $\pi/2$  electrical radians ahead of the direct axis and the model represents one pole pair of a four pole machine. The phases are arranged a, b, c, in counter clockwise fashion with time sequence abc. The power invariant transformations are:-

$$\mathbf{v}_{d} = \int_{\overline{3}}^{2} \left( \mathbf{v}_{a} \cos \theta + \mathbf{v}_{b} \cos(\theta - \frac{2\pi}{3}) + \mathbf{v}_{c} \cos(\theta - \frac{4}{3}\pi) \right)$$

$$\mathbf{v}_{q} = -\int_{\overline{3}}^{2} \left( \mathbf{v}_{a} \sin \theta + \mathbf{v}_{b} \sin(\theta - \frac{2}{3}\pi) + \mathbf{v}_{c} \sin(\theta - \frac{4}{3}\pi) \right)$$

If  $v_a = \sqrt{2} V \cos(\omega t + \alpha)$  the axis applied voltages become:

$$\mathbf{v}_{d} = \sqrt{3} \nabla \cos(\omega t + \alpha - \theta), \mathbf{v}_{q} = \sqrt{3} \nabla \sin(\omega t + \alpha - \theta)$$

The axis equations are:

$$\mathbf{v}_{dl} = (\mathbf{r}_{dl} + \mathbf{p}_{dl}^{L})\mathbf{i}_{dl} - \mathbf{L}_{ql}\Omega\mathbf{i}_{ql} + \mathbf{p}_{d}\mathbf{i}_{d2} - \mathbf{M}_{q}\Omega\mathbf{i}_{q2}$$

$$\mathbf{v}_{ql} = \mathbf{L}_{dl}\Omega\mathbf{i}_{dl} + (\mathbf{r}_{ql} + \mathbf{p}_{ql}^{L})\mathbf{i}_{ql} + \mathbf{M}\Omega\mathbf{i}_{d2} + \mathbf{p}_{q}\mathbf{i}_{q2}$$

$$\mathbf{0} = \mathbf{p}_{d}\mathbf{i}_{dl} + (\mathbf{r}_{d2} + \mathbf{p}_{d2}^{L})\mathbf{i}_{d2} \qquad (1)$$

$$0 = pM_q i_{q1} + (r_{q2} + pL_{q2})i_{q2}$$

The equations are rearranged to give:

$$p_{d1} = A_{11}v_{d1} + A_{13}i_{d1} + A_{14}\Omega_{q1} + A_{15}i_{d2} + A_{16}\Omega_{q2}$$

$$pi_{q1} = A_{22}v_{q1} + A_{23}\Omega i_{d1} + A_{24}i_{q1} + A_{25}\Omega i_{d1} + A_{26}i_{q2}$$

$$pi_{d2} = A_{31}v_{d1} + A_{33}i_{d1} + A_{34}\Omega i_{q1} + A_{35}i_{d2} + A_{36}\Omega i_{q2}$$

$$pi_{q2} = A_{42}v_{q1} + A_{43}\Omega i_{d1} + A_{44}i_{q1} + A_{45}\Omega i_{d2} + A_{46}i_{q2}$$
(2)

The A's are simply the set of resulting coefficients.

The equations of motion of the rotor and the load inertia are given by:

$$2 \left[ (\mathbf{L}_{d1} - \mathbf{L}_{q1}) \mathbf{i}_{d1} \mathbf{i}_{q1} - \mathbf{M}_{q} \mathbf{i}_{d1} \mathbf{i}_{q2} + \mathbf{M}_{d} \mathbf{i}_{q1} \mathbf{i}_{d2} \right]$$
  
=  $\mathbf{J}_{1} \mathbf{p} \mathbf{\Omega}_{1} + \mathbf{F} + \mathbf{K} (\mathbf{\Theta}_{1} - \mathbf{\Theta}_{2})$   
 $\mathbf{K} (\mathbf{\Theta}_{1} - \mathbf{\Theta}_{2}) = \mathbf{J}_{2} \mathbf{p} \mathbf{\Omega}_{2} + \mathbf{L}$  (3)

The factor 2 in the first of these equations is brought about by considering a 4-pole machine. In general this would be 2p where p = pole pairs.

At this stage a choice emerges between working in actual variables or adopting a per unit system which eliminates the dimensions of the machine from the analysis. The second course is followed here. The analogue computer depends on real time integration quite apart from fast or slow operation - and a perunit system in which real time is retained is easier to apply. However, while every quantity in the machine equations is replaced by its per unit quantity, certain terms require an additional factor. Per unit inductance for example is no longer dimensionless as it would be if time was put in per unit terms. It now has the dimensions of time.

Each quantity except time, including applied forces, voltages and other variables is replaced by per unit quantity = actual quantity/base quantity. Hence the base quantities must be considered. There are three fundamental base quantities, namely, base volts, base current and base speed. These are taken respectively to be rated volts per phase, rated current per phase of a comparative induction motor of the same frame size and iron dimensions, and the synchronous speed of a 4-pole 50-Hz machine.

The equations can be re-written in per unit form where the modified coefficients and per unit variables are represented by primed terms. Equations (2) and (3) become:

$$\mathbf{pi}_{d1}^{i} = \mathbf{A}_{11}^{i}\mathbf{v}_{d1}^{i} + \mathbf{A}_{13}^{i}\mathbf{i}_{d1}^{i} + \mathbf{A}_{14}^{i}\mathbf{\Omega}^{i}\mathbf{i}_{q1}^{i} + \mathbf{A}_{15}^{i}\mathbf{i}_{d2}^{i} + \mathbf{A}_{16}^{i}\mathbf{\Omega}^{i}\mathbf{i}_{q2}^{i}$$
(4)

and similarly for pit etc.

$$p\Omega_{1}^{i} = \frac{1}{J_{1}^{i}} \left[ A_{51}^{i} i_{d1}^{i} i_{q1}^{i} + A_{52}^{i} i_{d1}^{i} i_{q2}^{i} + A_{53}^{i} i_{q1}^{i} i_{d2}^{i} - F^{i} - K^{i} (\Theta_{1}^{i} - \Theta_{2}^{i}) \right]$$

$$p\Omega_{2}^{i} = \frac{1}{J_{2}^{i}} \left[ K^{i} (\Theta_{1}^{i} - \Theta_{2}^{i}) - L^{i} \right]$$
(5)

The A' values are tabulated in Appendix III. A small digital programme is used to evaluate numerical values.

Amplitude scaling is arranged to give 1 per unit current = 0.1 machine units on the computer (i.e. 10% of full output). Hence at least ten times normal current may be observed if required. Certain other gains require to be adjusted to give correct multiplication of the per unit quantities. The time scaling is lo0 times slower than real time.

An additional analogue programme is necessary to generate the applied axis voltages which change from 50-Hz to zero as the motor runs up from standstill to synchronism.

The equations are given by:

$$v_d = \sqrt{3} V \cos(\omega t + \alpha - \theta)$$

$$v_{\alpha} = \sqrt{3} V \sin(\omega t + \alpha - \theta)$$

Where  $\theta = \theta_0 + \int_0^t \Omega dt$  and  $\Omega$  is measured in

electrical radians per second.

### RESULTS

The programme has been designed to yield a comprehensive study of performance, particularly with regard to transient torques and run up, synchronisation and instability, as displayed by continuous oscillations of torque and speed in a limit cycle about the synchronous speed. The effects of variation in three basic resistance parameters, the reactance ratio and the load parameters (inertia, coupling stiffness and load torque) are brought out by means of a large number of freely-accelerating torque/speed curves. A selection of these is given in Figures 2 to 6.

#### Effect of Quadrature Axis Resistance (Figure 2)

Values of quadrature axis resistance that are either too high or too low promote instability. For D = 0.8, values of Q from 1 to rather more than 2 are permissible i.e. Q/D ratios from approximately 1 to 3 are possible with an optimum in the region of Q/D of 1.5.

### Effect of Direct Axis Resistance (Figure 3)

Varying direct axis resistance causes two effects simultaneously. a) Increasing D raises the general level of asynchronous torque, including low speed torque and b) increased D will promote instability whenever D exceeds Q, as has already been established from Figure 2.

Considering Figures 2 and 3 together, D may be raised to provide good run up, provided that Q remains greater than D. Too high values of both D and Q (i.e. each equal to or greater than 3) will produce severe subsynchronous speed oscillations - as indicated by the presence of loops instead of cusps in the torque/ speed curves.

#### Effect of Coupled Inertia (Figure 4)

The coupled inertia of 6 times rotor inertia causes very many torque oscillations to occur. This is a purely mechanical result to be expected. The high frequency oscillations appearing on the traces are due to the torsional effect of a normally proportioned mild steel shaft. Fig. 4c shows that if D and Q are too high, a normal coupled inertia will change the response from one in which subsynchronous speed oscillations take place to one in which there are permanent subsynchronous oscillations.

From Figures 2, 3 and 4 it may be concluded that the optimum values of D and Q for stability and fast run-up lie in the ranges: D = 1.5 to 2, Q/D approximately 1 to 1.5. As an estimate, the actual optimum values are: D = 1.5 and Q = 1.25 D.



Fig.	2	Effect tance.		of varying	ng ro	tor	quadrature	axis	resis-
		a Q	=	0.5	c	Q =	1.0	е	Q = 5
		bQ	=	0.8	d	Q =	2	f	Q = 10

# Effect of Stator Resistance (Figure 5)

Increasing the stator phase resistance by even a small amount causes small oscillations to commence. A 100% increase (S = 2) promotes complete instability. Reduction of stator resistance is beneficial and causes better synchronising with fewer dips in torque. Since motor heating will cause a 16% increase in winding resistance for a 40°C rise in copper temperature, the importance of amply proportioning the winding and providing adequate cooling is evident.

#### Effect of Reactance Ratio (Figure 6)

These responses show without question that too high a reactance ratio promotes instability, and that varying the rotor parameters has little effect. The reactance ratio of 5.2, moreover, is not unduly high in absolute terms, but even the near optimum values of D and Q, namely D = 1 and Q = 1.5 do not influence the response. The limit cycle takes a different shape from that arrived at with previous types of instability, e.g. kidney shaped as against elliptical. Only when the rotor direct axis resistance becomes high is the limit cycle broken, to be replaced by a subsynchronous limit cycle.



Fig. 3 Effect of varying rotor direct axis resistance Q = 1D = 1 d Q = 2D = 1 f Q = 3D = 1a e' Q = 2 D = 2 g Q = 3 D = 2ъ Q = 1 D = 1.5Q = 1 D = 2Q = 3D = 3C h

### PRACTICAL TORQUE SPEED CURVES

The method adopted to obtain practical torque/ speed curves was to mount the motor on a special stiff platform supported by special strain gauge straps. Such a method is extremely sensitive, but it is difficult to design a platform to have a completely linear frequency response over the whole frequency range under investigation (0-100 Hz). The actual torque at this frequency is in fact known to be somewhat less than the apparent measured value, and it should be assumed that the torque speed curve below half speed gives no more than approximate magnitudes. Above half speed, confidence can be placed in the results.

Figure 7 shows examples of recordings obtained from induction and reluctance motor rotors when placed









in a standard DlOOL metric stator. A four pole induction motor rated in this size at 3 hp would have a full load torque of 14.3 N.m (at 50-Hz). With a reluctance type rotor the full load torque would be about 10 N.m. The sequence shows the results of run-ups with three different rotors,

1)

A standard induction motor rotor. An axially laminated<sup>6</sup> reluctance motor rotor with 2) a saturated ratio of  $X_d$  to  $X_q$  of 3.7.

3) An axially laminated reluctance motor rotor with a saturated ratio of X<sub>d</sub> to X<sub>o</sub> of 5.2.

The effects of voltage variation, saturated reactance ratio, stator resistance and rotor resistances can be seen.



Fig. 6 Effect of increased reactance ratio a D = 1 Q = 1  $X_{d}/X_{q} = 5.2$ b D = 1.5 Q = 1

Figure 7a shows the run up of a standard induction motor against zero load and with no coupled inertia.

Figure 7b shows an axially laminated reluctance motor with a reactance ratio of 3.7 operating with a line to line voltage of 390 V (normal line volts).

Figure 7c shows the same motor with line voltage reduced to 310 V. Because of the reduced voltage, there is little saturation and X<sub>d</sub> has risen. The decreased stability that results can be seen by the longer time it takes to settle at the synchronous speed.

Figure 7d shows the effect of increasing the stator resistance of the same motor to twice the normal value whilst keeping the stator on reduced line volts of 310 V. The motor now operates in a continuous limit cycle about the synchronous point.

Figure 7e shows the effect of increasing the reactance ratio to 5.7 and lowering  $r_{q2}$  to about half its previous value. The adverse influences of increased reactance ratio and low  $r_{q2}$  has promoted instability, but more so the former. The line voltage is 390 V.

Figure 7f shows that when the line voltage is increased to 420 V the motor becomes stable. This is partly accounted for by the decrease in the value of  $X_d$  due to the increasing saturation.

Figure 7g should be compared with Figure 7e. It shows the effect of rotor resistance on the limit cycle. The line voltage is 390 V. The limit cycle has been reduced by decrease in rotor resistance. Rotor resistance was reduced by a factor of about 4:1 immersing the rotor in liquid nitrogen prior to re-inserting the rotor in the stator.

#### CONCLUSIONS

The results are reconcilable with those of Refs. 1-5 in as far as they may be compared with respect to the effect of the various parameter adjustments.

The measured responses give broad correlation with those obtained from the analogue computer - remarkably so in view of the approximations that have been made regarding the flux distribution in the actual machines. Clearly in the actual machine harmonics of both flux and current enter into the situation. General features of practical run up curves such as stability, degree of stability, broad torque magnitudes etc. are reconcilable with computed responses. In actual motors it has been found that stability is rather better by a small margin than that indicated by the computer study.

The torques predicted from these tests may be quite large. The asynchronous torque may maintain a



Fig. 7 Practical torque speed curves of reluctance motors.

level equal to full load torque, with peak values exceeding three times this figure.

It is clear from the above results that it is de-

sirable to keep the stator resistance as low as possible and that high reactance ratios will only give stable machines if the absolute values of the d and q rotor resistances are kept small and if the ratio of q and d axis resistance is somewhere around 1.5 to 1.

Present conclusions indicate that it is practical to identify the various kinds of instabilities and asynchronous performances that actual motors may show by means of a transient test, and that it is possible to diagnose the sources of these performances in terms of the magnitudes of certain critical parameters. Further refinement of the techniques described must depend on more sophisticated torque measuring apparatus.

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### APPENDIX I NOMENCLATURE

$v_a$ , $v_b$ , $v_c$ $i_a$ , $i_b$ , $i_c$	applied voltages to stator phases stator phase currents
idl, iql	axis currents in stator axis currents in rotor
r <sub>o</sub>	stator phase resistance: $rdl = r_{dl} = r_{el}$
$\mathbf{r}_{d2}$ , $\mathbf{r}_{q2}$	referred resistance of axis circuits on rotor
rl	stator phase resistance of comparative induction motor
r <sub>2</sub>	referred axis resistance of rotor of comparative induction motor
D	ratio rd2/r2
Q	ratio $r_{q2}/r_2$
S	ratio ra/rl
К	shaft stiffness
Jl	rotor inertia, kg.m <sup>-</sup>
J <sub>2</sub>	load inertia, kg.m <sup>2</sup>
F	friction and windage torque, N.m
L	load torque, N.m
θ	angle by which direct axis of rotor leads magnetic axis of stator phase a, electrical radians: position of rotor with respect to stator reference
$\theta_1 \\ \theta_2$	above angle, mechanical radians position of load inertia with respect to stator reference mechanical radians
Ω	rotor speed. electrical radians/s
Ω	rotor speed, mechanical radians/s
<u>ດວ່</u>	load speed, mechanical radians/s
ພ້	supply angular frequency

APPENDIX II BASE QUANTITIES

base volts/phase  $V_B = 225 V$ 

base current/phase  $I_B = 4.7 A$ base resistance  $R_B = V_B/I_B = 47.8 \Omega$  (also  $L_B$  and  $X_B$ ) base power  $P_B = 3V_BI_B = 3170 \text{ W} = 4.25 (3 \text{ hp} = 0.706 \text{ pu})$ base mechanical speed  $\Omega_{1B} = 50\pi \text{ rad/s}$ base electrical speed  $\Omega_{\rm B} = 100\pi$  rad/s base torque  $T_B = P_B / \Omega_{1B} = 20.2$  N.m (14.3 N.m for 3 hp at 50m rad/s) base inertia  $J_B = P_B / \Omega_{1B}^2 \times 1 \text{ s} = 0.129 \text{ kg.m}^2$ base shaft stiffness =  $T_B / 50\pi = 0.129 \text{ N.m/rad}$ 

base angle =  $50\pi$  rad

Note that the above definition of base inertia gives pu inertia equal to twice the inertia constant (H) as nor-mally defined, times  $s^{-1}$ .

$$A_{11}^{i} = \frac{-L_{d2}^{i}}{(M_{d}^{2} - L_{d1}L_{d2})^{i}} \qquad A_{12}^{i} = 0$$

$$A_{13}^{\prime} = -r_{d1}^{\prime}A_{11}^{\prime} = -Sr_{1}^{\prime}A_{11}^{\prime} \qquad A_{14}^{\prime} = L_{q1}^{\prime}\Omega_{B}A_{11}^{\prime}$$

$$A_{15}^{i} = \frac{r_{d2}^{i} M_{d}^{i} A_{11}^{i}}{L_{d2}^{i}} = \frac{Dr_{2}^{i} M_{d}^{i} A_{11}^{i}}{L_{d2}^{i}} \qquad A_{16}^{i} = M_{q}^{i} \Omega_{B} A_{11}^{i}$$

$$A_{21}^{\prime} = 0$$
  $A_{22}^{\prime} = \frac{-u_{q2}^{\prime}}{(M_{q}^{2} - L_{q1}L_{q2})^{\prime}}$ 

$$A_{23}^{i} = -L_{d1}^{i}\Omega_{B}A_{22}^{i}$$
  $A_{24}^{i} = -r_{q1}^{i}A_{22}^{i} = -Sr_{1}^{i}A_{22}^{i}$ 

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$$A_{25}^{*} = -M_{d}^{*}\Omega_{B}A_{22}^{*} \qquad A_{26}^{*} = \frac{r_{q2}^{*}M_{q}^{*}A_{22}^{*}}{L_{q2}^{*}} = \frac{Qr_{2}^{*}M_{q}^{*}A_{22}^{*}}{L_{q2}^{*}}$$

$$A_{31}^{i} = \frac{M_{d}^{i}}{(M_{d}^{2} - L_{d1}L_{d2})^{i}} \qquad A_{32}^{i} = 0$$

$$A_{33}^{i} = -r_{d1}^{i}A_{31}^{i} = -Sr_{1}^{i}A_{31}^{i}$$
  $A_{34}^{i} = L_{q1}^{i}\Omega_{B}A_{31}^{i}$ 

$$A_{35}^{i} = \frac{r_{a2}^{i}a_{a1}^{i}A_{31}^{i}}{M_{d}^{i}} = \frac{\mu r_{2}^{i}a_{a1}^{i}A_{31}^{i}}{M_{d}^{i}} \qquad A_{36}^{i} = M_{q}^{i}\Omega_{B}A_{31}^{i}$$

$$A_{41}^{*} = 0$$
  $A_{42}^{*} = \frac{m_{q}}{(M_{q}^{2} - L_{q1}L_{q2})^{*}}$ 

$$A_{43}^{i} = -L_{d1}^{i}\Omega_{B}A_{42}^{i} \qquad A_{44}^{i} = -r_{q1}A_{42}^{i} = -Sr_{1}^{i}A_{42}^{i}$$

$$A_{43}^{i} = -r_{q1}A_{42}^{i} = -Sr_{1}^{i}A_{42}^{i} \qquad A_{44}^{i} = -r_{q1}A_{42}^{i} = -Sr_{1}^{i}A_{42}^{i}$$

$$A_{45}^{*} = -M_{d}^{*} B_{42}^{*} \qquad A_{46}^{*} = -\frac{M_{q}^{*}}{M_{q}^{*}} = -\frac{M_{q}^{*}}{M_{q}^{*}}$$
 $A_{51}^{*} = (L_{d1}^{*} - L_{q1}^{*}) \frac{\Omega_{B}}{3} \qquad A_{52}^{*} = -M_{q}^{*} \frac{\Omega_{B}}{3}$ 

 $A_{53}^{I} = M_{d}^{I} \frac{\Omega_{B}}{2}$ 

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